Indirect Taxation in Durable Goods Markets

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Abstract

We consider the effects of indirect taxation in durable goods markets. First, we show that social welfare is higher under ad valorem taxation than under unit taxation; this advantage is increasing in durability in case of renting, while an increase in durability has an ambiguous effect in case of selling. Second, taxation may increase profits when the good is sold due to a mitigation of the commitment problem. Third, unit taxation implies an inefficiently high durability in case of renting, but counteracts the phenomenon of “planned obsolescence” in case of selling; an ad valorem tax is neutral concerning durability.

Keywords: Durable Goods, Monopoly, Taxation, Time Consistency

JEL classification: D4, H2, L1

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1 Introduction

Indirect taxation (also called as commodity taxation) is an important source of revenue for governments around the world. The defining characteristic of an indirect tax is the creation of a wedge between the price that (final) purchasers actually pay and the price that the producers actually receive. There are two basic forms of commodity taxes: an ad valorem tax, charged as a proportion of the product price, or a per unit tax (also called as specific tax), charged as a fixed amount per unit. Both types of indirect taxes are employed in practice. The most common examples of an ad valorem tax are sales taxes (e.g., multiple levels of government in the United States impose sales taxes), and value added taxes (e.g., European countries generally impose a value added tax). Various countries around the world also levy unit taxes on durable goods in the form of excise taxes, i.e., taxes that apply to particular products. For example, many countries charge a vehicle registration and a vehicle license tax that is fixed.\(^1\)

It is therefore relevant to understand if and how the choice between both taxes alters market outcomes. Understanding how the form of an indirect tax affects market behavior and outcomes in durable goods markets may help in designing better public policy.

The comparison between ad valorem and unit taxes is one of the main issues in public economics. There is an extensive literature on the effects of indirect taxation. It is a well-known result that unit and ad valorem taxes (when appropriately chosen) are equivalent in perfectly competitive markets. It is also well established that an ad valorem tax is welfare superior to an equal-yield unit tax in both monopoly and oligopoly markets. However, these findings are restricted to non-durable goods. In this paper, we explore the effects of commodity taxation in durable goods markets. At the beginning of our analysis, we examine whether the welfare superiority of ad valorem taxation holds in a durable good setting and how the welfare difference between ad valorem and unit taxation is affected when the product’s durability is increasing. However, the main focus of our paper is on market power and its policy implications and thus refers to the

\(^1\)In some countries, the license tax is based on the engine size, the fuel type or the vehicle’s carbon dioxide emissions, e.g., Germany, Ireland, Japan, and United Kingdom. Thus, the vehicle license tax is a fixed amount per unit for each vehicle category.
central issues in industrial organization. We are especially interested in the effects of taxation on commitment power and how the durability choice is affected by both tax forms. To the best of our knowledge, no previous work has systematically analyzed the effects of different forms of indirect taxation in the context of durable goods.

We show that the welfare superiority of ad valorem taxation holds in durable goods markets—and is increasing in durability when the good is rented out. In other words, the higher the product’s durability, the more market outcomes under ad valorem and unit taxation differ. In case of selling, however, an increase in durability has an ambiguous effect on the social welfare and tax revenue difference. Ad valorem taxation still welfare dominates unit taxation but this relative advantage may diminish in the product’s durability.

Further, we show that an increase in tax rates may increase profits in the case of selling when the commitment problem concerning future output is intense.

Concerning durability levels in a renting monopoly, we show that unit taxation implies an inefficiently high level of durability, whereas ad valorem taxation leads to an efficient level of durability. Moreover, we demonstrate that unit taxation counteracts the firm’s incentive to practice planned obsolescence in case of selling whereas ad valorem taxation does not.

In terms of policy recommendations, our results indicate a trade-off for a social planner. On the one hand, ad valorem taxation is preferable for a given durability. On the other hand, unit taxation leads to a higher level of durability than ad valorem taxation. Higher durability may be welfare enhancing when the production is attended by negative externalities, e.g., pollution or if the firm gets more market power by practicing planned obsolescence.

The remainder of the paper is organized as follows: In Section 2, we present the related literature. The basic model is presented in Section 3. A comparison of revenues and social welfare for both tax schemes follows in Section 4. In Section 5, we analyze the effects of ad valorem and unit taxation on profits. In Section 6, we investigate the influence of taxation on the durability choice. A Conclusion follows.
2 Related Literature

This paper is related and contributes to two strands of literature. First, we build on the literature concerned with indirect taxation (see Keen, 1998, for a survey of this literature). The main conclusion is that in perfectly competitive markets unit and ad valorem taxes are equivalent (when chosen appropriately). It is also well established that an ad valorem tax is associated with a lower deadweight loss compared to a unit tax in both monopoly and oligopoly markets with the same tax revenue. In case of a monopoly, Skeath and Trandel (1994) have even shown that, for any unit tax, there exists an ad valorem tax that results in a lower price, higher profits, and higher tax revenues. Consequently, ad valorem taxation Pareto dominates specific taxation in case of a single producer. In general, ad valorem taxation welfare-dominates specific taxation in oligopolistic markets, too (see, e.g., Delipalla and Keen, 1992; Cremer and Thisse, 1994; and Hamilton, 1999). However, Anderson et al. (2001) have shown that this may not be the case in price competition with differentiated products.

Second, our paper is related to the literature on durable goods (see Waldman, 2003, for a survey of this literature). Durability is one of the main characteristics of many goods and prompts a number of specific issues. For instance, one major issue is concerned with consumer expectations and time inconsistency problems as introduced by Coase (1972). How are current prices and the choice of other variables affected by a firm’s future actions? Coase conjectured that the output decisions of a monopolistic seller may not maximize the firm’s overall profit. The seller has an incentive to sell too much output units in the future if he is unable to commit himself to a certain output path. Rational consumers expect a future price reduction and adjust their willingness to pay for today’s products accordingly. A second set of questions is concerned with durability choice and the related issue of planned obsolescence (see Bulow, 1982, 1986). In particular, how does time inconsistency affect the durability choice?

Our paper is also related to the literature on emissions taxation in durable goods markets (see, e.g., Goering and Boyce, 1996 and 1999; Runkel, 2002; and Driskill and Horowitz, 2007).
However, they did not systematically explore the difference between ad valorem and unit taxation in a general manner.

3 The Model

We employ the standard two-period durable goods framework introduced by Bulow (1982, 1986). This means that there are two discrete periods of time: first period (“today”) and second period (“tomorrow”) which allows us to use backward induction; all qualitative results extend to finitely many periods. For simplicity, we consider a monopolistic market. Oligopoly is probably a more accurate description for most durable goods markets; nevertheless, it is market power, not strict monopoly, that drives our results.

We denote the product’s durability by $0 < \delta \leq 1$. If the firm’s period one output is perfectly durable, then $\delta = 1$; otherwise part of the first-period production perishes. If first-period production is $q_1$, the total quantity of first-period units available for use in the second period is $\delta q_1$. We initially consider $\delta$ to be exogenous. This assumption is only made to simplify the exposition and is relaxed in Section 6.

All agents in the market have perfect and complete information. Further, we assume that there is a perfect second-hand market for the surviving $\delta q_1$ units in the second period. This is a common assumption in the literature. Alternatively, we could have assumed that there exists no secondhand market for the good. Both assumptions are used in the literature (see, e.g., Sobel, 1991, Karp and Perloff, 1996, Waldman, 1996). Our qualitative results do not depend on this assumption.

Without loss of generality, we disregard discounting.

The inverse (rental) demand for the durable good in each period depends on the total available quantity, i.e.,

$$ p_1 = f(q_1), $$

$$ p_2 = f(\delta q_1 + q_2), $$
where $f : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, strictly decreasing, and concave.

Costs are given by $C_1(q_1)$ in the first period, and $C_2(q_2)$ in the second period, and we assume that $C'_i > 0$, and $C''_i \geq 0$, $i \in \{1, 2\}$.

A durable goods monopolist faces the problem of competing with a future incarnation of himself: By selling the product today, he reduces future demand. This implies lower prices in future periods. Consumers anticipate any price reductions and adjust their willingness to pay for today’s products accordingly. Since the consumers can hold back on purchases today, the uncommitted seller has to offer lower prices than a seller with commitment power. Without commitment, the resulting outcome with a continuum of consumers converges to the outcome under perfect competition when the time horizon is infinite (see Coase, 1972, and Gul et al., 1986). The central problem of the durable goods monopolist is thus his commitment power. There are several instruments that can be employed to at least partially create commitment. The main suggestion for the monopolist is to rent out the product rather than to sell it. However, renting is not always feasible or desirable. For instance, moral hazard and adverse selection problems may arise. We will thus consider both cases: renting and selling without commitment. The solution concept is that of subgame perfect Nash equilibrium.

We first consider the case of selling ($S$). To ensure a subgame perfect Nash equilibrium, we have to begin our analysis in the second period. In the second period, first-period output is given. So the monopolist’s problem in period two is

$$\max_{q_2} \left\{ p_2 (\delta q_1 + q_2) q_2 - C_2(q_2) \right\}. \quad (1)$$

The corresponding first-order condition is

$$p_2 (\delta q_1 + q_2) + p'_2 (\delta q_1 + q_2) q_2 - C'_2(q_2) = 0. \quad (2)$$

Consumers anticipate the behavior of the monopolist. The price that buyers of the durable good are willing to pay in the first period also depends on their expectation of the price in the second
period—as a perfect resale market exists in the second period in which the surviving $\delta q_1$ units bought during the first period may change hands when the market price exceeds the value an owner attaches to the good. Therefore, the first-period price in the case of selling is

$$p_1^S \equiv p_1 (q_1) + \delta p_2 (\delta q_1 + q_2).$$

The optimization problem of the monopolist is given by

$$\max_{q_1, q_2} \pi^S \equiv [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2)] q_1 - C_1 (q_1) + p_2 (\delta q_1 + q_2) q_2 - C_2 (q_2), \quad (3)$$

but subject to (2).

In case of renting ($R$), the monopolist’s optimization problem is

$$\max_{q_1, q_2} \pi^R \equiv [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2)] q_1 - C_1 (q_1) + p_2 (\delta q_1 + q_2) q_2 - C_2 (q_2),$$

which is identical to (3). However, since the renting firm has no commitment problem concerning its future output, its optimization problem is not constrained by condition (2).

## 4 Tax Revenues and Social Welfare

This section compares ad valorem and unit taxation in terms of tax revenues and social welfare. Our approach follows Suits and Musgrave (1953) and Anderson et al. (2001). To simplify the exposition, we assume constant average cost, $c > 0$, in both periods; all results extend readily to a case with a more general cost function. We first consider the case of a renting monopolist. Under a unit tax $t_U$, the monopolist keeps an amount of $p_i - t_U$ of the price $p_i$ paid by the consumers, $i \in \{1, 2\}$. His profit can then be written as

$$\pi^R_{t_U} \equiv [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2) - c_U] q_1 + [p_2 (\delta q_1 + q_2) - c_U] q_2, \quad (4)$$
with effective cost $c_U \equiv c + t_U$.

Under an ad valorem tax $t_V$, the monopolist’s producer price is $(1 - t_V) p_1$. Profit under the ad valorem tax is then given by

$$\pi_R^V \equiv (1 - t_V) \left[ (p_1 (q_1) + \delta p_2 (\delta q_1 + q_2) - c_V) q_1 + (p_2 (\delta q_1 + q_2) - c_V) q_2 \right],$$  

with effective cost $c_V \equiv \frac{c}{1 - t_V}$.

When tax rates are set such that $c_U = c_V$, i.e., when $t_U = \frac{t_V}{1 - t_V} c$, the two profit functions are identical except for the factor $(1 - t_V)$. This factor acts like a pure profit tax, implying that it is neutral since it does not affect firm behavior. Thus, when taxes are set so that $t_U = \frac{t_V}{1 - t_V} c$, equilibrium prices and quantities will be the same under the two taxes. This benchmark allows us to easily compare tax revenues across the taxes for the same level of social welfare.

The tax revenue $(T)$ collected under the ad valorem tax in the case of renting is

$$T^R_V = t_V \left[ (p_1 (q^*_1) + \delta p_2 (\delta q^*_1 + q^*_2)) q^*_1 + p_2 (\delta q^*_1 + q^*_2) q^*_2 \right],$$

where $q^*_i$ denotes the equilibrium quantity in period $i$. The tax revenue collected under a unit tax in the case of renting is

$$T^R_U = t_U (q^*_1 + q^*_2) = \frac{t_V}{1 - t_V} c (q^*_1 + q^*_2).$$

We can now compare the tax revenues under the two different tax schemes, given $t_U = \frac{t_V}{1 - t_V} c$. The tax revenue difference ($\triangle$) is
\[ \Delta^R \equiv T^R_V - T^R_U = t_V \left[ \left( (p_1 q_1^* + \delta p_2 (\delta q_1^* + q_2^*)) - \frac{c}{1 - t_V} \right) q_1^* + \left( p_2 (\delta q_1^* + q_2^*) - \frac{c}{1 - t_V} \right) q_2^* \right] \]

\[ = t_V \left[ p (q_1^*) q_1^* + (\delta q_1^* + q_2^*) p_2 (\delta q_1^* + q_2^*) - \frac{c}{1 - t_V} (q_1^* + q_2^*) \right]. \]

(6)

Note that the only difference in equation (6) to the equilibrium profit under the ad valorem tax in equation (5) lies in the factors \((1 - t_V)\) and \(t_V\), respectively. Hence \(\Delta^R \geq 0\) (\(\Delta^R = 0\) in the case of no production). In other words, for any given unit tax \(t_U\), there exists an ad valorem tax rate \(t_V\) that yields the same social welfare with a higher tax revenue. Since the tax revenue difference for the rental and the sales case differ only in their equilibrium quantities, the same results hold for the sales case, too.

In addition, we can show that \(\Delta^R\) is increasing in the level of durability, that is, in \(\delta\). A renter will offer the same quantity in both periods, that is, \(q_1^* = \delta q_1^* + q_2^*\) or \((1 - \delta) q_1^* = q_2^*\), equal to the quantity in a static model. An increase in \(\delta\) will reduce \(q_2^*\) and thus reduce the term \(\frac{c}{1 - t_V} (q_1^* + q_2^*)\), while the term \(p (q_1^*) q_1^* + (\delta q_1^* + q_2^*) p_2 (\delta q_1^* + q_2^*)\) that represents cumulative revenues will remain unchanged. This implies that \(\Delta^R\) is increasing in \(\delta\).

Since equilibrium quantities are the same under ad valorem and unit taxation if \(t_U = \frac{t_V}{1 - t_V} c\), the sum of tax revenues and profits must be the same, too. So we are able to conclude that the relative advantage of an ad valorem tax concerning the level of tax revenues is increasing in \(\delta\), but concerning profits the relative disadvantage of the ad valorem tax is increasing in \(\delta\), too. The higher \(\delta\), the more the market outcomes under ad valorem and unit taxation differ, provided that the durable good is rented out and \(t_U = \frac{t_V}{1 - t_V} c\).

In the case of selling, however, the effect of an marginal increase in \(\delta\) on the tax revenue difference \((\Delta^S)\) is ambiguous. Contrary to the rental case, \(q_1^* = \delta q_1^* + q_2^*\) does not hold anymore. The tax revenue difference may increase or decrease in \(\delta\), depending on the level of
product durability and the form of the inverse demand function.\textsuperscript{2}

Provided that there is an equilibrium with positive quantities, we can summarize the above results in the following

**Lemma.** For any given unit tax rate, there exists an ad valorem tax rate $t^*$ that yields higher tax revenues for the same level of social welfare, irrespective of whether the durable good is sold or rented out. In case of renting, the difference in tax revenues is increasing in durability, while an increase in durability has an ambiguous effect on the tax revenue difference in the case of selling.

Since the inverse (rental) demand function is continuous and has a negative first and second derivative, we can immediately follow from the Lemma that a marginal decrease in the ad valorem tax rate (below $t^*$) implies a higher level of social welfare and a higher tax revenue than under the given unit tax rate. We can thus state the following

**Proposition 1.** There exists an ad valorem tax rate that implies both more tax revenue and higher output levels for any equilibrium under a given unit tax rate, irrespective of whether the good is sold or rented out. In case of renting, this relative advantage of ad valorem taxation is increasing in durability, whereas this may not be the case with selling.

The first part of Proposition 1 is consistent with the findings for non-durable goods and implies that a government that either seeks to maximize tax revenues or social welfare or any weighted average of both prefers ad valorem taxation. The second part, however, introduces a new element specific to durable goods: the advantage of ad valorem taxation is increasing in the durability of the good in the case of renting; this may not be true in the case of selling. The intuition is that an increase in durability has no effect on the total quantity in both periods (i.e., an increase in $\delta$ increases $q_1$ and reduces $q_2$) if the good is rented out, while the change in total quantities is ambiguous in the case of selling, implying that an increase in $\delta$ may increase or reduce the advantage of ad valorem taxation over unit taxation.

\textsuperscript{2}A detailed proof can be obtained upon request from the authors.
The next section analyzes how commodity taxation influences profits.

5 The Effects of Taxation on Profits

5.1 Rental Case

If the firm rents its product, it owns the entire stock of output and is not constrained by consumers’ expectations of future pricing behavior. We will first consider the case of a unit tax. Let $\Pi^R_U$ denote the maximum profit function in the case of renting and a unit tax, that is,

$$\Pi^R_U \equiv \max_{q_1,q_2} \left\{ \left[ p_1(q_1) + \delta p_2 (\delta q_1 + q_2) - t_U \right] q_1 - C(q_1) + \left[ p_2 (\delta q_1 + q_2) - t_U \right] q_2 - C(q_2) \right\}.$$ 

We can prove the following

**Proposition 2.** An increase in the unit tax rate cannot raise the profit of a renting monopolist.

*Proof.* The envelope theorem implies that

$$\frac{d\Pi^R_U}{dt_U} = -(q_1^{R*} + q_2^{R*}) < 0,$$

and $\frac{d\Pi^R_U}{dt_U} = 0$ in the case of no production. \[\square\]

Now consider the case of an ad valorem tax and let $\Pi^R_V$ denote the maximum profit function in the case of renting, that is,

$$\Pi^R_V \equiv \max_{q_1,q_2} \left\{ \left(1 - t_V\right) \left[ p_1(q_1) + \delta p_2 (\delta q_1 + q_2) \right] q_1 - C(q_1) + \left(1 - t_V\right) p_2 (\delta q_1 + q_2) q_2 - C(q_2) \right\}.$$ 

Again, the envelope theorem implies the following

**Proposition 3.** An increase in the ad valorem tax rate cannot raise the profit of a renting monopolist.
Proof. Using the envelope theorem, we can show that

\[
\frac{d\Pi^R}{dt_V} = - \left[ \left( p_1 (q_1^{R*}) + \delta p_2 (\delta q_1^{R*} + q_2^{R*}) \right) q_1^{R*} + p_2 (\delta q_1^{R*} + q_2^{R*}) q_2^{R*} \right] < 0,
\]

and \( \frac{d\Pi^R}{dt_V} = 0 \) in the case of no production.

The findings in this subsection merely reflect the standard non-durable goods result that the tax imposes costs but has no corresponding benefit to the firm, since there are no commitment problems with buyers. In other words, there is no time inconsistency problem. Only in cases where the firm sells its output without commitment a commodity tax could have a positive effect on profit by increasing commitment power. To demonstrate this, we now turn to the case of an uncommitted seller.

5.2 Sales Case

We first consider the case of a unit tax. In the second period, the firm faces the optimization problem

\[
\max_{q_2} \pi_2 = p_2 (\delta q_1 + q_2) q_2 - C_2 (q_2) - t_U q_2.
\]

The corresponding first-order condition is

\[
p_2' (\delta q_1 + q_2) q_2 + p_2 (\delta q_1 + q_2) - C_2' (q_2) - t_U = 0.
\]  (7)

Let \( \Pi^S_U \) denote the maximum profit function in the case of selling and a unit tax, that is,

\[
\Pi^S_U \equiv \max_{q_1,q_2} \left\{ [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2) - t_U] q_1 - C (q_1) + [p_2 (\delta q_1 + q_2) - t_U] q_2 - C (q_2) \right\}
\]

subject to (7).

We can now state the following
Proposition 4. An increase in the unit tax may increase profits of a selling monopolist.

Proof. The application of the Envelope Theorem for constrained optimization yields \( \frac{d\Pi}{dt_V} > 0 \) if

\[ \lambda > q_1^{S*} + q_2^{S*}, \]

where \( \lambda \) denotes the Lagrange multiplier of the associated constrained optimization problem. Otherwise, taxation decreases profit (or may have no marginal effect on profit).

We now consider the case of an ad valorem tax. Once again, we start our analysis in the second period. When the second period arrives, the firm’s optimization problem is

\[ \max_{q_2} \pi_2^S = (1 - t_V) p_2 (\delta q_1 + q_2) q_2 - C_2 (q_2). \]

The first-order condition is

\[ (1 - t_V) \left[ p_2' (\delta q_1 + q_2) q_2 + p_2 (\delta q_1 + q_2) \right] - C_2' (q_2) = 0. \tag{8} \]

Let \( \Pi_V^S \) denote the maximum profit function in the case of selling and an ad valorem tax, that is,

\[ \Pi_V^S \equiv \max_{q_1, q_2} \{(1 - t_V) [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2)] q_1 - C (q_1) + (1 - t_V) p_2 (\delta q_1 + q_2) q_2 - C (q_2)\} \]

subject to (8).

We can now state the following

Proposition 5. An increase in the ad valorem tax may increase profits of a selling monopolist.

Proof. Using the Envelope Theorem for constrained optimization, it is straightforward to show
that \( \frac{d\Pi}{dV} > 0 \) if
\[
\lambda > \frac{\left[ p_1 (q_1^{S*}) + \delta p_2 (\delta q_1^{S*} + q_2^{S*}) \right] q_1^{S*} + p_2 (\delta q_1^{S*} + q_2^{S*}) q_2^{S*}}{p_2 (\delta q_1^{S*} + q_2^{S*}) q_2^{S*} + p_2 (\delta q_1^{S*} + q_2^{S*})}
\]
or
\[
\lambda > \frac{\text{Total revenue in both periods}}{\text{Marginal revenue in the second period}}
\]

where \( \lambda \) denotes the Lagrange multiplier of the associated constrained optimization problem again. Otherwise, taxation decreases profit (or may have no marginal effect on profit).

The last two propositions imply that profit may even increase with higher tax rates in the case of selling if \( \lambda \), the “shadow price of commitment”, is sufficiently high and \( q_1^{S*} \) and \( q_2^{S*} \) are positive (i.e., an interior solution exists). In other words, commodity taxation may have the effect of increasing market power. The intuitive explanation behind this perverse fact is that taxation may help the monopolist to mitigate his commitment problem with the buyers concerning future output decisions. An increase in the tax rate reduces the second-period output—and the overall output—but increases the second-period price which lessens the incentive of consumers to delay their purchase. This implies the following effects: First, since some consumers do not delay their purchase anymore, this may lead to a positive quantity effect in period one. Second, buyers in the first period adjust their willingness to pay accordingly because they expect the future price reduction to be less sharp than in case of no tax increase. This may lead to a positive price effect on the first-period profit. Thus, the first-period profit may increase and overcompensate the reduction in the second-period profit which implies a higher overall profit. So, taxation increases overall profits in situations when the commitment problem is particularly intense, i.e., when \( \lambda \) is high enough.\(^3\)

\(^3\)The results are in line with the findings of Goering and Boyce (1996) and Goering (2012). Goering and Boyce (1996) proved that it is possible for a durable goods monopolist to be made better off under a per unit
6 Endogenous Durability

The previous sections assumed that durability is exogenously determined. We relax this assumption in this section. The cost function in the first period now incorporates $\delta$ as an additional variable, that is, total costs in the first period are $C_1(q_1, \delta)$. We assume that $C_1(q_1, \delta)$ is twice continuously differentiable with a positive first derivative and a non-negative second derivative in both arguments. Our analysis implies that the optimal level of durability is characterized by the following conditions:

(i) Rental case and ad valorem taxation:

The firm equates the marginal cost of durability per unit—so that one more of the first-period units will still be used in the second period—with the marginal cost of production in the second period. This is also the socially efficient level of durability. Ad valorem taxation implies an efficient level of durability, since it does not affect the cost functions in both periods.

(ii) Rental case and unit taxation:

The firm equates the marginal cost of durability per unit with the marginal cost of production in the second period plus the amount of the unit tax that has to be paid for the marginal unit produced in the second period. This results in an inefficiently high level of durability, as unit taxation effectively increases production cost in the future, which implies that increasing durability becomes more profitable.

emissions tax. However, their results are driven by the assumption that the per unit emissions in the second period are larger than in the first period. Goering (2012) showed that the overall profit of a durable goods monopolist may increase if an ad valorem tax is levied only in period one using a highly stylized model (i.e., linear demand and constant marginal costs).
(iii) Sales cases:

There is an incentive to reduce durability (i.e., to practice planned obsolescence) in order to reduce the commitment problem in the case of selling. Ad valorem taxation still has no effect on cost functions, implying that durability is inefficiently low due to planned obsolescence. Unit taxation effectively increases production cost in the second period, implying that it counteracts the incentive for planned obsolescence. The equilibrium level of durability may be inefficiently low or high depending on whether the “planned obsolescence effect” dominates the “unit taxation effect” or not.

The formal proofs are given in the following subsections.

6.1 Rental Case

The optimization problem of a renting monopolist under ad valorem taxation is

\[
\max_{q_1, q_2, \delta} \pi^R_{V} = (1 - t_V) \left[ p_1(q_1) + \delta p_2(\delta q_1 + q_2) \right] q_1 + (1 - t_V) \left[ p_2(\delta q_1 + q_2) \right] q_2 - C_1(q_1, \delta) - C_2(q_2).
\]

Thus, we can state the following

**Proposition 6.** Ad valorem taxation leads to an efficient level of durability in case of renting.

**Proof.** The first-order conditions are

\[
\frac{\partial \pi^R_{V}}{\partial q_1} = (1 - t_V) \left[ p_1(q_1) + p'_1(q_1) q_1 + \delta p'_2(\delta q_1 + q_2) (\delta q_1 + q_2) + \delta p_2(\delta q_1 + q_2) \right] - \frac{\partial C_1}{\partial q_1} = 0, \quad (9)
\]

\[
\frac{\partial \pi^R_{V}}{\partial q_2} = (1 - t_V) \left[ p'_2(\delta q_1 + q_2) (\delta q_1 + q_2) + p_2(\delta q_1 + q_2) \right] - C'_2 = 0, \quad (10)
\]

\[
\frac{\partial \pi^R_{V}}{\partial \delta} = (1 - t_V) q_1 \left[ p'_2(\delta q_1 + q_2) (\delta q_1 + q_2) + p_2(\delta q_1 + q_2) \right] - \frac{\partial C_1}{\partial \delta} = 0. \quad (11)
\]
Combining the last two conditions yields

\[
\frac{1}{q_1} \frac{\partial C_1}{\partial \delta} = C'_2. \tag{12}
\]

Condition (12) implies that the firm will choose an efficient level of durability and corresponds to the case without taxation, as shown in Bulow (1986). Thus, the ad valorem tax is neutral in terms of the durability choice.

The problem of a renting monopolist in case of a unit tax is

\[
\max_{q_1, q_2, \delta} \pi^R_U = [p_1(q_1) + \delta p_2(\delta q_1 + q_2) - t_U] q_1 + [p_2(\delta q_1 + q_2) - t_U] q_2 - C_1(q_1, \delta) - C_2(q_2). \tag{13}
\]

We can state the following

**Proposition 7.** Unit taxation leads to an inefficiently high level of durability in case of renting.

**Proof.** The first-order conditions of the maximization problem in (13) are

\[
\frac{\partial \pi^R_U}{\partial q_1} = p_1(q_1) + \frac{\partial p_1}{\partial q_1} q_1 + \delta p_2(\delta q_1 + q_2) - t_U + \frac{\partial p_2}{\partial q_1} (\delta q_1 + q_2)(\delta q_1 + q_2) - \frac{\partial C_1}{\partial q_1} = 0, \tag{14}
\]

\[
\frac{\partial \pi^R_U}{\partial q_2} = p'_2(\delta q_1 + q_2)(\delta q_1 + q_2) + p_2(\delta q_1 + q_2) - t_U - C'_2 = 0, \tag{15}
\]

\[
\frac{\partial \pi^R_U}{\partial \delta} = q_1 \left( p'_2(\delta q_1 + q_2)(\delta q_1 + q_2) + p_2(\delta q_1 + q_2) \right) - \frac{\partial C_1}{\partial \delta} = 0. \tag{16}
\]

Combining the last two conditions implies

\[
\frac{1}{q_1} \frac{\partial C_1}{\partial \delta} = C'_2 + t_U. \tag{17}
\]

Equations (12) and (17) only differ in \( t_U > 0 \) and so the renting firm will choose an inefficiently high level of durability under a unit tax.\(^4\)

\(^4\)Goering and Boyce (1999) have done a similar analysis for an emissions tax. They stated that an emissions
6.2 Sales Case

When the second period arrives, the monopolist faces the following optimization problem under unit taxation:

$$\max_{q_2} \pi_2^S = p_2 (\delta q_1 + q_2) - C_2 (q_2) - t_U q_2.$$  

The first-order condition is

$$p_2' (\delta q_1 + q_2) q_2 + p_2 (\delta q_1 + q_2) - C_2' (q_2) - t_U = 0. \quad (18)$$

In the first period, the problem of the monopolist is

$$\max_{q_1, q_2, \delta} \pi_U^S = \left[ p_1 (q_1) + \delta p_2 (\delta q_1 + q_2) - t_U \right] q_1 + \left[ p_2 (\delta q_1 + q_2) - t_U \right] q_2 - C_1 (q_1, \delta) - C_2 (q_2), \quad (19)$$

but subject to the constraint in (18). Calculating $\partial \pi / \partial \delta$ and taking into account that $dq_2/d\delta q_1$ is implicitly determined by the constraint in (18), yields the following condition on optimal durability:

$$\frac{1}{q_1} \frac{\partial C_1}{\partial \delta} = C_2' + t_U + \delta q_1 p_2' (\delta q_1 + q_2) \frac{d (\delta q_1 + q_2)}{d \delta q_1}. \quad (20)$$

The term $d (\delta q_1 + q_2) / d\delta q_1$ is generally positive (i.e., if the demand curve is more steeply downward sloping than the marginal cost curve; see Bulow, 1986, on pp. 734–735). This implies that the term $\delta q_1 p_2' (\delta q_1 + q_2) \frac{d (\delta q_1 + q_2)}{d \delta q_1}$ is negative and thus resulting in an effect that lowers durability (below the efficient level). This last term explains the phenomenon of “planned obsolescence,” as described by Bulow (1986). However, a unit tax creates a countervailing effect by increasing the level of durability represented by $t_U > 0$.

We now consider the effect of an ad valorem tax. The optimization problem is

$$\max_{q_2} \pi_2 = (1 - t_U) p_2 (\delta q_1 + q_2) q_2 - C_2 (q_2).$$

tax will induce a renting firm to increase product durability if emissions are a function only of output. However, Runkel (2002) proved that some of the results in Goering and Boyce (1999) are not valid.
The first-order condition is

\[
(1 - t_V) \left[ p'_2 (\delta q_1 + q_2) q_2 + p_2 (\delta q_1 + q_2) \right] - \frac{dC_2}{dq_2} = 0. \tag{21}
\]

The problem of the monopolist in the first period is

\[
\max_{q_1, q_2, \delta} \pi^S = (1 - t_V) [p_1 (q_1) + \delta p_2 (\delta q_1 + q_2)] q_1 + (1 - t_V) p_2 (\delta q_1 + q_2) q_2 - C_1 (q_1, \delta) - C_2 (q_2) \tag{22}
\]

but subject to the constraint in (21). This leads to the following condition on optimal durability

\[
\frac{1}{q_1} \frac{\partial C_1}{\partial \delta} = C'_2 + \delta q_1 p'_2 (\delta q_1 + q_2) \frac{d(\delta q_1 + q_2)}{d\delta q_1}. \tag{23}
\]

The only difference between (20) and (23) lies in \(t_U\). In other words, the ad valorem tax is neutral concerning the durability choice. We can state the following

**Proposition 8.** Unit taxation leads to a higher level of durability compared to ad valorem taxation in case of selling.

To summarize, ad valorem taxation does not influence marginal cost of production whereas under unit taxation, second-period units become more expensive compared to an increase in durability of the first-period units. This implies that unit taxation leads to an inefficiently high level of durability in the case of renting but countervails the effect of planned obsolescence in the case of selling.

The results on endogenous durability indicate that a social planner prefers ad valorem taxation in the case of renting. On the contrary, in the case of selling, a unit tax may have an welfare increasing effect since it may offset the effect of planned obsolescence that may serve as an commitment device for future output paths of the monopolistic seller. Unit taxation may also be ecologically worthwhile to facilitate the sustainable use of production resources.
7 Conclusion

This paper has analyzed the effects of ad valorem taxation and unit taxation in durable goods markets. We have shown that for any given unit tax rate, there exists an ad valorem tax rate that yields a higher level of social welfare, irrespective of whether the goods are being sold or rented out. The welfare advantage of ad valorem taxation even increases with product durability in case of renting. This indicates that the higher the product’s durability, the more the market outcomes under ad valorem and unit taxation differ. In case of selling, however, an increase in durability has an ambiguous effect on the difference of social welfare between ad valorem and unit taxation.

Further, we have demonstrated that profits may increase in the case of selling as taxation mitigates the commitment problem.

Concerning durability levels, we have shown that unit taxation implies a higher level of durability than under no taxation and ad valorem taxation. This is because a unit tax makes second-period units more expensive compared to an increase in durability of the first-period units. Thus, in case of renting—the case in which the firm has full commitment power—the durability level is inefficiently high under a unit tax. In case of selling, however, the effect of a unit tax on the durability choice countervails the incentive of a monopolistic seller to practice planned obsolescence. In addition, we have proven that ad valorem taxation is neutral concerning durability for both renting and selling and implies an efficient durability choice in case of renting, but implies an inefficiently low level of durability due to planned obsolescence.

This article is a first step towards a deeper understanding of the impact of commodity taxation in durable goods markets. For example, on the empirical side, there is a need for more micro-level evidence on the effects of commodity taxation.
References


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