A Better Asymmetric Model of Changing Volatility in Stock Returns: Trend-GARCH

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Abstract

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In this paper we consider the theoretical and empirical relevance of a new family of conditionally heteroskedastic models with a trend dependent conditional variance equation: the Trend-GARCH model. The interest in these models lies in the fact that modern microeconomic theory often suggests the connection between the past behavior of time series, the subsequent reaction of market individuals, and thereon changes in the future characteristics of the time series. Our results reveal important properties of these models, which are consistent with stylized facts in financial data sets. They can also be employed for model identification, estimation, and testing. The empirical analysis of a broad variety of asset prices significantly supports the existence of trend effects. The Trend-GARCH model proves to be superior to alternative models such as EGARCH, AGARCH, TGARCH or GARCH-in-Mean in replicating the leverage effect in the conditional variance and in fitting the news impact curve.

JEL: C22, C52, G12

Keywords: GARCH, trend, volatility, news impact curve;
1 Motivation

Innovations in economic time series analysis have mainly been driven by the wish to explain stylized facts that puzzled the older models. This paper aims to connect two well known stylized facts, the asymmetry of returns and the importance of technical trading, in order to develop a new class of GARCH models, the Trend-GARCH model. The results reveal important properties of these models, which are consistent with stylized facts in financial time series.

The volatility feedback effect (Campell and Hetschel (1992)) has been used to explain the presence of conditional leftskewedness observed in stock returns through an increase in future volatility following all kinds of news. However, markets amplify the impact of bad news but dampen the impact of good news on returns. This typically results in the conditional leftskewedness of returns. The news impact curve (NIC)(Engle and Ng (1993)) of such an asset price series is thus asymmetric. Several extensions of the GARCH model – e.g. EGARCH, AGARCH, TGARCH, or GARCH-M – catch this specific stylized fact of financial time series. The Trend-GARCH model proves superior to these alternatives for two reasons. Firstly, it fits better to the empirical data (see section 3), e.g. with respect to the asymmetric NIC and the significance of trend effects on future volatility. Secondly, the structure of the model is the result of a microeconomic investment decision model.

Current strands of economic literature also include heterogeneous traders in microstructure models of asset markets. Typical types of traders are

- **fundamentalists** who react on fundamental analysis and **chartists** who base their decisions on technical analysis (see e.g. Lux (1995)),

- **noise traders** (see e.g. De Long, Shleifer, Summers and Waldmann (1990)) who react to market fads and disturbed information,

- **dealers** (market makers) who coordinate the initial buying and selling orders, clear the market and extract information from order flow and **traders** who make the initial orders.

These models may explain well known stylized facts of asset prices such as fat tails, bubbles, herd behavior, etc. The overwhelming majority of these models use at least one type of traders who follows a positive feedback trading rule. These traders may be approximated by a simple trend-following trading rule. Bauer and Herz (2004) show in
the context of an exchange rate model that the presence of such traders in the market increases the conditional volatility. The same argument is valid for prices of arbitrary assets.

The remainder of the paper introduces the Trend-GARCH model and compares it to alternative extensions of the GARCH model (section 2.1). The news impact curve of this model class is then compared to other GARCH models (section 2.2). Section 3 presents the empirical evidence. Section 4 concludes.

2 GARCH extensions and the Trend-GARCH model

2.1 The models

Following Engle (1982) we use the standard notation for the innovations of a discrete time real-valued conditional heteroskedastic stochastic process \( \{ \varepsilon_t \} \),

\[
\varepsilon_t = z_t \sigma_t,
\]

where \( E_t (z_t) = 0 \) and \( VAR_t (z_t) = 1 \), and the conditional variance \( \sigma_t^2 \) is a positive time-varying and measurable function with respect to the information set \( I_{t-1} \) available at time \( t - 1 \). \( E_t (\cdot) \) and \( VAR_t (\cdot) \) denote the expectation and variance operator conditional on the information set \( I_{t-1} \).

In the original ARCH(p) model, Engle (1982) defines the conditional variance \( \sigma_t^2 \) as a linear function of the lagged squared innovations. To allow for more flexibility in modelling the variance structure than the simple Markovian dependency up to lag \( p \) of the squared innovations in the ARCH model, Bollerslev (1986) proposes the more general GARCH(p,q) model. The conditional variance \( \sigma_t^2 \) in the GARCH model is a linear function of the lagged squared innovations and the lagged conditional volatilities

\[
\sigma_t^2 = \omega + \alpha (L) \varepsilon_t^2 + \beta (L) \sigma_t^2.
\]

\( L \) denotes the backshift operator and \( \alpha (L) \equiv \sum_{i=1}^{p} \alpha_i L^i \) and \( \beta (L) \equiv \sum_{i=1}^{q} \beta_i L^i \). Given the usual constraints, like finiteness of the fourth moment \( E (\varepsilon_t^4) < \infty \), the conditional variance process can be rewritten as ARMA(max \( (p,q) \)) in the squared innovations

\[
[1 - \alpha (L) - \beta (L)] \varepsilon_t^2 = \omega [1 - \beta (L)] (\varepsilon_t^2 - \sigma_t^2).
\]

The process is stable and covariance stationary if all roots of \( 1 - \alpha (L) - \beta (L) \) and \( \beta (L) \) lie outside the unit circle.
This type of time series processes has been expanded in many ways. Typically economic time series can be characterized by a long memory property of the volatility process, i.e. the volatility clusters are more compact than one would expect by standard GARCH-processes. This observation leads typically to GARCH parameter estimates which imply a non stationary volatility process or are very close to the stationarity border. To account for these problems Engle and Bollerslev (1986) proposed the IGARCH and Bollerslev, Baillie and Mikkelsen (1996) the FIGARCH models. In Integrated GARCH\(_{(p,i,q)}\) processes the volatility process is integrated of order \(i\) indicated by \((1-L)^i\) in the corresponding equation. In Fractionally Integrated GARCH\(_{(p,d,q)}\) processes the volatility process is fractionally integrated of order \(d\), i.e. the process of the conditional volatility is defined by

\[
\sigma_t^2 = \omega [1 - \beta (1)]^{-1} + \left\{ 1 - [1 - \beta (L)]^{-1} (1 - L)^{d} \phi (L) \right\} \varepsilon_t^2. \tag{3}
\]

The power series representation of \((1-L)^d\) gives an ARCH(\(\infty\)) characterization of (3) (see Bollerslev et al. (1996)).

A second type of generalization of the original model incorporates the asymmetry of economic time series. A stylized characteristic of financial markets is their asymmetric reaction to good and bad news. The negative correlation between past returns and future volatility is called leverage effect. Engle and Ng (1993) investigate the leverage effect in stock markets quantitatively. Zakoian (1994) and Glosten, Jagannathan and Runkle (1993) use a dummy variable for days with negative innovations to model the conditional volatility. This model is known as the TGARCH (treshold GARCH) or GJR model:

\[
\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2, \quad d_{t-1} = \begin{cases} 
1 & \text{if } \varepsilon_{t-1} < 0 \\
0 & \text{if } \varepsilon_{t-1} \geq 0
\end{cases}.
\]

Alternative models which account for the leverage effect are e.g. the EGARCH model of Nelson (1991)

\[
\ln \sigma_t^2 = \omega + \beta \ln \left( \sigma_{t-1}^2 \right) + \alpha \left| \varepsilon_{t-1} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{4}
\]

and the AGARCH model of Engle (1990)

\[
\sigma_t^2 = \omega + \alpha (\varepsilon_{t-1} - \gamma)^2. \tag{5}
\]

The leverage effect accounts partly for the asymmetric behavior of the conditional variance process of economic time series. The different effects of good and bad news
depend on their effects on the trends in the markets.\footnote{This aspect is not realized in GARCH-in-Mean models since they only model the impact of the conditional volatility on the location parameter of the innovation.}

Another family of extensions of the GARCH model account for the impact of the actual volatility on the expected future mean. The ARCH-M models were introduced by Domowitz and Hakkio (1985) and Engle, Lilien and Robins (1987). The generalized models are denoted GARCH-in-Mean or GARCH-M models. While leaving the equation for the conditional variance unchanged, the mean equation is supplemented by a linear function of the conditional variance or standard deviation. For a typical GARCH-M model with ARMA mean equation this yields the set of equations

\begin{align}
\sigma^2_t &= \omega + \alpha (L) \varepsilon^2_t + \beta (L) \sigma^2_t \\
x_t &= \omega_m + \alpha_m (L) \varepsilon^2_t + \beta_m (L) x_t + \gamma f (\sigma^2_t). \tag{6a}
\end{align}

where $f(x)$ is $x$ or $\sqrt{x}$. The in-Mean concept may be mixed with other GARCH extensions (see e.g. the GRJ-GARCH-M in Lanne and Saikkonen (2004)). The empirical literature on the influence of the conditional variance on the mean process is voluminous, but not unanimous. Lanne and Saikkonen (2004) find a positive relation between risk and return within a GARCH-M setup not until applying a z-distribution for the conditional distribution of the innovations. The quantitative effects are small compared to the trend effects presented here.

Asset prices in general, such as stock prices, exchange rates or derivatives, are the result of supply and demand, i.e. of the behavior of traders and investors. A number of studies (e.g. Menkhoff (1998)) affirm the influence of technical analysis on the decisions of traders. Technical analysis usually relies on the interpretation of some sort of trend measure.\footnote{Some types of technical analysis, such as candle sticks, are exceptions to that rule.} Theoretical models like De Long et al. (1990) or Bauer and Herz (2004) show that technical analysis is rational in an heterogeneous environment and may influence the variance of the return process.

The trend following character of technical trading results in more technical induced activity and volatility if trends are large.\footnote{Only few approaches to technical trading, such as adverse selection or constant portfolio weights, do not induce trend following.} Thus the conditional volatility is not only affected by the sign of the innovation, but also by its effect on a trend. If an innovation $\varepsilon_t$ amplifies the currently observed trend, market activity and volatility increase. If an innovation dampens the currently observed trend, market activity and volatility decrease.
Since trends can be positive or negative, the influence of the sign of the innovation depends on the current trend.

Bauer and Herz (2004) suggest to analyze a new variant of GARCH models which account for the effects of trends to volatility. The Trend-GARCH(s,p,q) model is characterized by the volatility process

$$\sigma_t^2 = \omega + \beta (L) \sigma_t^2 + \alpha (L) \varepsilon_t^2 + \gamma \left( \frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i} \right)^2. \quad (7)$$

The trend component is simply the total increment of the last s periods. This trend measure is typically used to proxy trend estimates in technical trading models (compare Lux and Marchesi (2000)). Alternatively, an exponential trend can be used. The exponential-Trend-GARCH(s,p,q) with parameter $\lambda$ model is characterized by the volatility process

$$\sigma_t^2 = \omega + \beta (L) \sigma_t^2 + \alpha (L) \varepsilon_t^2 + \gamma \left( \frac{1-\lambda^s}{1-\lambda} \sum_{i=1}^{s} \lambda^{i-1} \varepsilon_{t-i} \right)^2. \quad (8)$$

One might be tempted to generalize this model class using a conditional volatility process like

$$\sigma_t^2 = \omega + \beta (L) \sigma_t^2 + \alpha (L) \varepsilon_t^2 + \gamma (L) \varepsilon_t^2, \quad (9)$$

where $\tilde{\gamma}$ is a polynomial of degree s. However, the nonlinear structure of $(\tilde{\gamma} (L) \varepsilon_t)^2$ and the potential identification problems with the coefficients of $\alpha$ indicate severe difficulties for the applicability of such a model. On the other hand, the formulation of the Trend-GARCH volatility process given in equation 11 shows that the trend is more than an external variable determining the conditional variance. As the trend depends on the innovations just as the variance does and both influence the size of future innovations, the conditions determining the stationarity properties of the time series differ from the conditions for ordinary GARCH models. The task of a rigorous analysis of these stationarity conditions is left to future research. Simulations based on coefficients estimated from the economic time series analyzed in this article behave stable and may be assumed to have a stationary volatility process.

Furthermore, the trend component can not be replaced by switching to a larger time scale. For example a Trend-GARCH(20,p,q) model (20 working days approximate one month) is not equivalent to a GARCH(p,q) model on monthly data. The Trend-GARCH(20,p,q) model on daily data measures the influence of the monthly trend on
daily conditional volatility while the GARCH(p,q) model on monthly data measures the monthly conditional volatility.

Finally the differences of the GARCH-M models to the Trend-GARCH models will be summarized. Trend-GARCH models differ significantly from GARCH-in-Mean models within two respects. Firstly, while both classes of models constitute a relation between the first and second moment of the innovations, they are exact opposites with respect to the direction of this relation. Trend-GARCH models are based on the influence of the empirical (observed) past mean of the innovations on the future volatility, while GARCH-M models represent an impact of the actual volatility on the expected future mean. Secondly, the relation of trend and conditional variance/standard deviation in GARCH-M models is linear. However, the empirical data suggests a U-shaped relation, i.e. a strong trend of either sign causes a rise in future volatility (see section 3). The leverage effect merely accounts for the asymmetry of this dependence. This behavior cannot be modeled within a GARCH-M setup, since the trend linearly depends on a function of the variance.

As we will show in the empirical section (section 3), the Trend-GARCH model is superior in explaining and representing the empirical data, i.e. the U-shaped relation of current trend and future volatility and the fit of the NIC. It also partially accounts for the long memory property of the volatility process in stock returns in a (Trend-)FIGARCH setup.

2.2 The news impact curve

The news impact curve (NIC) relates today’s returns to tomorrow’s volatility. Following Engle and Ng (1993):

"The news impact curve is the functional relationship between conditional variance at time $t$ and the shock term (error term) at time $t-1$, holding constant the information dated $t-2$ and earlier, and with all lagged conditional variance evaluated at the level of the unconditional variance."

It is thus the appropriate measure for the model comparison in this paper. Engle and Ng (1993) defined it as the expected conditional variance of the next period conditional

\footnote{For exchange rate time series the leverage effect should depend on the inhomogenity of the market micro structure. The exchange rate between two similar countries will be effected symmetricly by news as good news for one half of the traders is bad news for the other half and vice versa.}
on the current shock \( \varepsilon_t \)

\[ E \left( \sigma_{t+1}^2 | \varepsilon_t \right) . \]  

(10)

For the classical ARCH and GARCH models the NIC is a parabola with minimum at \( \varepsilon_t = 0 \). The GARCH-M model shows the same NIC as the corresponding basic not-in-Mean model, since this extension only influences the mean equation. The NICs of the asymmetric model of Engle (1990) (see equation 5) and the EGARCH model of Nelson (1991) have their minimum at \( \varepsilon_t = 0 \), too. However, in these cases the NIC is not symmetric around 0, but skewed. Negative news drive volatility up more than good news. In these models, any news today drive up volatility tomorrow.

The NIC of the AGARCH model of Engle (1990) is not symmetric around 0, either. In this model the NIC is a right-shifted parabola. This potentially suggests slightly positive news as a requirement for the markets to remain as calm as possible. 'No news' in this model implies a higher volatility than in tranquil markets.

In contrast to the former models, where the location of the NIC is uniquely determined through the model parameters, the Trend-GARCH models yield a NIC which depends on the current trend. Based on equation (9) a simple calculation shows that for Trend-GARCH models

\[
E \left( \sigma_{t+1}^2 | \varepsilon_t \right) = \omega + \beta \left( L \right) \sigma_{t+1}^2 + \alpha \left( L \right) \varepsilon_t^2 + \left( \gamma \left( L \right) \varepsilon_t \right)^2 \\
= \omega + \beta \left( L \right) \sigma_{t+1}^2 + \alpha_1 \varepsilon_t^2 + \alpha^* \left( L \right) \varepsilon_t^2 + \left( \tilde{\gamma}_1 \varepsilon_t + \tilde{\gamma}^* \left( L \right) \varepsilon_t \right)^2,
\]

(11)

where \( \alpha^* \left( L \right) = \alpha \left( L \right) - \alpha_1 L \) and \( \tilde{\gamma}^* \left( L \right) = \tilde{\gamma} \left( L \right) - \tilde{\gamma}_1 L \) denote the respective polynomials without the linear term. For a Trend-GARCH(s,1,1) model equation (11) simplifies to

\[
E \left( \sigma_{t+1}^2 | \varepsilon_t \right) = \omega + \beta \sigma_t^2 + \alpha \varepsilon_t^2 + \left( \tilde{\gamma}_1 \varepsilon_t + \sum_{i=1}^{s-1} \tilde{\gamma}_i \varepsilon_{t-i} \right)^2,
\]

If the trend is positive then slightly negative news, which slow the trend down, calm the market. If the trend is negative then slightly positive news generate a relatively tranquil market. Also, the minimum level of future volatility depends on the size of the current trend. Strong trends have two implications on future volatility. Firstly, the stronger the trend, the higher the future volatility, i.e. the minimum of the NIC increases with the absolute size of the trend. Secondly, larger trends require stronger signals to be weakened. To tranquilize the market an innovation has to be the larger the stronger the trend.

The economic intuition behind this analysis is based on the existence of a positive feedback trading rule of some of the market participants. The positive feedback trading
rule implies a trend following behavior. The larger the trend, the more traders react. Since traders may take short and long positions, or as an exchange rate trend always is a positive trend for one and a negative trend for the other country, the influence of the trend is comparatively symmetric. The trend model is more flexible in reflecting the actual market’s situation than the pure leverage models.

Figures 1 and 2 give an overview about the NIC of the selection of models presented here. A GARCH(1,1) specification is used for each alternative model. Figure 2 shows the Trend-GARCH NICs while figure 1 displays the alternative models. The influence of the current volatility $\sigma_t$ on the NIC is like an additive constant (multiplicative for the EGARCH model), i.e. different values may only shift the NIC on the vertical axis.\(^5\) Table 1 contains the corresponding parameter values and formulas for the NICs and the plots for the alternative models and the Trend-GARCH-model with null, positive and negative trend.

3 Empirical evidence

The empirical analysis is separated into four parts. In a first step, the Trend-GARCH model is fitted to the data. The estimates of the trend parameter are highly significant for all but one asset price series. In the second step, all GARCH models are estimated on various asset prices. Here the Trend-GARCH model has the highest explanatory value for the conditional volatility. In the third step the dependence of trend and variance in the data is revealed by a kernel regression. Finally, the analyses of a Trend-FIGARCH model shows that the trend accounts for a part of the long memory property of the volatility process in the analyzed time series.

The data cover daily opening stock prices of six major US companies in the DJII (HomeDepot, IBM, Coca Cola, Johnson & Johnson, Procter & Gamble, and Exxon), four stock indices (Dax, DJII, Hang Seng, and Nikkei), and two US Dollar exchange rates (to the Hungarian Forint and the Euro).\(^6\) Data run from 01/01/1990 to 15/07/2004.\(^7\)

\(^5\) Different values for the past conditional volatilities and the past squared returns in larger GARCH(p,q) models may only shift the NIC on the vertical axis.

\(^6\) The choice of the six companies from the DJII sample was random.

\(^7\) The Hungarian Forint series starts at 06/17/1993 and the Euro series starts at 01/01/1999.
<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter values</th>
<th>Plotted formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1) and GARCH-M</td>
<td>$\beta = 0.9, \sigma_i = 1, \omega = 0.1, \alpha = 1$</td>
<td>$y = x^2 + 1$</td>
</tr>
<tr>
<td>AGARCH</td>
<td>$\beta = 0.9, \sigma_i = 1, \omega = 0.1, \alpha = 1, a = 5$</td>
<td>$y = (x - \frac{1}{2})^2 + 1$</td>
</tr>
<tr>
<td>TGARCH</td>
<td>$\beta = 0.9, \sigma_i = 1, \omega = 0.1, \alpha = 1, a = 5$</td>
<td>$y = x^2 + \frac{1}{2} x I_{x &lt; 0} + 1$</td>
</tr>
<tr>
<td>EGARCH</td>
<td>$\beta = 0.9, \sigma_i = 1, \omega = 0.1, \alpha = 1, a = 5$</td>
<td>$y = \exp(x + \frac{1}{2}</td>
</tr>
<tr>
<td>Trend-GARCH: no trend</td>
<td>$\beta = 0.75, \tilde{\gamma_i} = 0.5, \tilde{\gamma_i} = 0.5, \sum_{i=1}^{s-1} \tilde{\gamma_i} \tilde{\sigma}_{t-i} = 1$</td>
<td>$y = x^2 - x + 1$</td>
</tr>
<tr>
<td>Trend-GARCH: positive trend</td>
<td>$\beta = 0.75, \tilde{\gamma_i} = 0.5, \tilde{\gamma_i} = 0.5, \sum_{i=1}^{s-1} \tilde{\gamma_i} \tilde{\sigma}_{t-i} = 1$</td>
<td>$y = x^2 - x + 1$</td>
</tr>
<tr>
<td>Trend-GARCH: negative trend</td>
<td>$\beta = 0.75, \tilde{\gamma_i} = 0.5, \tilde{\gamma_i} = 0.5, \sum_{i=1}^{s-1} \tilde{\gamma_i} \tilde{\sigma}_{t-i} = 1$</td>
<td>$y = x^2 - x + 1$</td>
</tr>
</tbody>
</table>
3.1 Trend-GARCH estimation

As a first step in the empirical analysis, the Trend-GARCH model is fitted to the data. The mean equation of the GARCH model resembles a random walk according to the standard assumption of perfect markets. The variance equation is amended by a squared trend term.

\[
\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 + \gamma \left( \frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i} \right)^2
\]

\[
= \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 + \gamma (\text{trend}_{t-1})^2
\]

The trend is estimated as the increase of the asset price in the past five observations, i.e. a Trend-GARCH(5,1,1) is estimated. The estimation is performed with the GARCH
routine implemented in the SPLUS program package. The trend is calculated a priori and treated as an exogenous variable.

Table 2 gives the results of the estimates of the squared trend parameter $\gamma$ in the variance equation.

The estimates of the trend parameter are positive for all series. They are highly significant for all but the Hungarian Forint series. The Trend-GARCH model describes the data better than the pure GARCH-Model.

Table 3 gives the results of the estimates of the squared trend parameter in the variance equation according to equation (12) from simulated data. The parameters for the simulation are estimated from the original DJII price series. Data is simulated for each of the alternative models: GARCH(1,1), AGARCH(1,1), TGARCH(1,1), EGARCH(1,1), and GARCH-M(1,1). The estimates of the trend parameters are not significant for any of the models. Therefore these models do not reveal the volatility structure of the true data, since they don’t catch the trend dependence of the volatility process.
Table 2: Estimates of the parameter $\gamma$ of the squared trend in the variance equation

| Time series       | Estimate of $\gamma$ | Std. Error | t-value | Pr($>|t|)$ |
|-------------------|-----------------------|------------|---------|-----------|
| HomeDepot         | 1.34                  | 0.06       | 23.03   | 0.00      |
| IBM               | 1.42                  | 0.08       | 17.98   | 0.00      |
| Coca Cola         | 2.01                  | 0.06       | 33.42   | 0.00      |
| Johnson & Johnson | 1.07                  | 0.08       | 13.02   | 0.00      |
| Procter & Gamble  | 1.07                  | 0.06       | 16.89   | 0.00      |
| Exxon             | 1.27                  | 0.06       | 19.94   | 0.00      |
| Dax               | 1.02                  | 0.05       | 18.57   | 0.00      |
| DJII              | 0.70                  | 0.06       | 11.66   | 0.00      |
| Hang Seng         | 1.33                  | 0.05       | 28.79   | 0.00      |
| Nikkei            | 1.85                  | 0.07       | 26.43   | 0.00      |
| HUF/USD           | 0.29                  | 0.18       | 1.57    | 0.12      |
| EUR/USD           | 1.55                  | 0.10       | 14.74   | 0.00      |

Table 3: Estimates of the parameter $\gamma$ of the squared trend in the variance equation from simulated data

| Simulated data from model | Estimate of $\gamma$ | Std. Error | t-value | Pr($>|t|)$ |
|---------------------------|-----------------------|------------|---------|-----------|
| GARCH(1,1)                | 0.015                 | 0.014      | 1.05    | 0.15      |
| AGARCH                    | 0.015                 | 0.018      | 0.84    | 0.20      |
| TGARCH                    | 0.0087                | 0.018      | 0.49    | 0.31      |
| EGARCH                    | 0.019                 | 0.020      | 0.94    | 0.17      |
| GARCH-M                   | 0.03                  | 0.028      | 1.3     | 0.10      |

3.2 NIC estimation

The news impact curves of the five GARCH models presented in section 2.2 are estimated on the empirical data. The NIC is directly connected to the conditional variance of the time series. This leads to the following problem when comparing alternative models. The true conditional variance of the asset price $\rho$ is unknown. The estimation of each of the models – as a side effect – yields an estimation of the conditional variance, too. These estimated conditional variances differ from model to model. Estimates of the conditional volatility resulting from one of the GARCH-models by definition fulfill the variance equation of the respective model. Using one of these estimates for the model comparison would bias the
outcome in favor of the model whose variance estimate was used. In order to circumvent this complication, a model free estimation of the variance is used: the empirical variance of the past s innovations $\varepsilon_t = p_t - p_{t-1}$ with $s = 3$.

$$\hat{\sigma}_t^2 = \sum_{i=0}^{s} \varepsilon_{t+i}^2 - \left( \sum_{i=0}^{s} \varepsilon_{t-i} \right)^2$$  \hspace{2cm} (13)

When using this proxy for the conditional variance as the dependent variable in the regressions for the NIC, another problem emerges. The windows from which the dependent variable $\varepsilon_{t+1}$ and the independent $\sigma_t^2$ are calculated would overlap. In order to estimate the NIC of the five models the future conditional variance is therefore proxied by the square of the next innovation $\varepsilon_{t+1}^2$. This is in line with the common interpretation of the NIC that describes the influence of the actual innovation on the size of the future innovations. Table 4 gives the five regression equations.

For the Trend-GARCH model the trend is estimated as the increase of the asset price over the past three observations.

The regression is performed by OLS. Table 5 displays the $r^2$ of each estimate. Clearly the Trend-GARCH model is superior. Each of the four GARCH modifications has the same number of estimated parameters and the explanatory value of the Trend-GARCH model is larger than that of any of the other models for each time series.

For all estimates the signs of the coefficients are either as expected or the estimates are insignificant. The trend component in the Trend-GARCH model is positive and highly significant for all 12 time series.\textsuperscript{8}

\textsuperscript{8}The complete list of all estimated parameters in all models and all time series is available upon request from the author.
3.3 Kernel regressions

Another model free regression shows the relationship between the trend and the future conditional variance. Trend and conditional variance are estimated by the mean and the empirical variance of the first differences of the log asset prices within windows of length 5, i.e.

\[
\text{trend}_t = \frac{1}{s} \sum_{i=0}^{s} \varepsilon_{t-i}
\]

\[
\hat{\sigma}_t^2 = \sum_{i=0}^{s} \varepsilon_{t-i}^2 - \left( \sum_{i=0}^{s} \varepsilon_{t-i} \right)^2,
\]

with \(s = 5\) and \(\varepsilon_t = 100(p_t - p_{t-1})\). The first difference of the logarithmic asset price is upscaled to obtain percentage returns.

To resemble the time structure of the model, data points \((\text{trend}_t, \hat{\sigma}_t^2)\) are analyzed, i.e. the impact of the current trend on future volatility. Now a kernel regression with a Gauss-kernel is performed on these data points and simultaneous confidence bands around
the kernel regressions are constructed using a bootstrap type estimator. At the given significance level of 0.05 the probability that the estimated relation does not leave the confidence band at any point is 0.95. Since the windows overlap, the estimates, which are close in time, are correlated ($\alpha$-mixing). The kernel regression is justified by the amount of data points, since we can asymptotically neglect this type of dependencies between the data points. Also for the estimation of the confidence bands we can asymptotically neglect these dependencies using an bootstrap algorithm of Neumann and Polzehl (1998).

Bauer and Herz (2004) show on several examples that regressions on simulated GARCH and FIGARCH data do not result in the U-shaped dependencies between current trend and future volatility which appears in the regressions on the original data. The same holds true for simulations on the GARCH extensions analyzed here.\footnote{The graphs of kernel regressions on simulated data are available from the author upon request.}

3.4 Estimation of the parameter of fractional integration

Finally, the comparison of the estimates from FIGARCH and Trend-FIGARCH models on the sample time series yields the following results. The dependence of the future conditional volatility on the current observed trend also accounts for a part of the long memory property of the volatility process in the analyzed time series. For each of the Trend-FIGARCH models the partial integration parameter is estimated significantly lower than in the standard FIGARCH setup. However, the fractional integration remains significant. Also, the estimate for the squared trend parameter was positive and significant in each time series.\footnote{Bauer and Herz (2005) show similar results for all 6 series in their sample of USD exchange rates of industrialized countries.} Table 7 gives the results of the estimations of the parameters of fractional integration in FIGARCH and Trend-FIGARCH models with their standard errors, the difference of these estimates and the p-value for the difference. Also, estimates of the squared trend parameters in the variance equation of the Trend-FIGARCH models with their standard errors are presented.

4 Summary

This paper presents the Trend-GARCH model. The model is defined at the outset. Then we give an economic interpretation and motivation for this type of time series. We compare
the model to alternative GARCH extensions, such as EGARCH, AGARCH, TGARCH, and GARCH-M with respect to their variance equations and the news impact curves.

Finally, we illustrate the empirical relevance of the Trend-GARCH model on a sample of 12 asset prices (6 major US companies, 4 stock indices, and 2 US Dollar exchange rates). Trend-GARCH models may explain the empirical dependence between current trend and future variance in economic time series. Neither GARCH-in-Mean models (which reflect the influence of current conditional volatility on future trend) nor asymmetric GARCH extensions such as TGARCH, EGARCH or AGARCH account for these empirical facts. The Trend-GARCH model proves to be superior to these alternative models in replicating the leverage effect in the conditional variance and in fitting the news impact curve, and accounts for a part of the long memory property of the volatility process in the analyzed time series.

References


Table 6: Kernel regressions of past trend on future conditional volatility and simultaneous confidence bands at 5% level

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<tr>
<th>Company</th>
<th>Trend</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>HomeDepot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coca-Cola</td>
<td></td>
<td></td>
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<tr>
<td>Johnson &amp; Johnson</td>
<td></td>
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<tr>
<td>Procter &amp; Gamble</td>
<td></td>
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<tr>
<td>Exxon</td>
<td></td>
<td></td>
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<tr>
<td>HUF/USD</td>
<td></td>
<td></td>
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<tr>
<td>EUR/USD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dax</td>
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<tr>
<td>DJII</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hangsen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nikkei</td>
<td></td>
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</table>
Table 7: Estimates of the parameter of fractional integration in FIGARCH and Trend-FIGARCH models and estimate of the squared trend parameter in the variance equation

<table>
<thead>
<tr>
<th>Time series</th>
<th>$d_1$: estimate of $d$ in FIGARCH model</th>
<th>Std. Error of $d_1$</th>
<th>$d_2$: estimate of $d$ in Trend-FIGARCH model</th>
<th>Std. Error of $d_2$</th>
<th>$d_1 - d_2$</th>
<th>p-value of $d_1 - d_2$</th>
<th>estimate of $\gamma$ in Trend-FIGARCH model</th>
<th>Std. Error of $\gamma$</th>
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<tbody>
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<td>0.02</td>
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<td>0.11</td>
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<td>0.61</td>
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<td>0.13</td>
<td>0.01</td>
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<td>0.11</td>
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<td>Nikkei</td>
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<td>0.67</td>
<td>0.04</td>
<td>0.16</td>
<td>9.1·10^{-5}</td>
<td>0.53</td>
<td>0.02</td>
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<td>HUF/USD</td>
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<td>0.15</td>
<td>0.02</td>
<td>0.69</td>
<td>1.1·10^{-3}</td>
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<td>0.08</td>
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<td>EUR/USD</td>
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<td>0.18</td>
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<td>1.37</td>
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